

ANALOG PROGRAMMABLE ELECTRONIC CIRCUIT-BASED CHAOTIC LORENZ SYSTEM

¹Fadhil Rahma, ²Ramzy S. Ali, ³Luigi Fortuna

^{1,2}Department of Electrical Engineering, College of Engineering,
University of Basrah,
Basrah, Iraq,

{fadhil.rahma, ramzy.salim}@uobasrah.edu.iq

³ Dipartimento di Ingegneria Elettrica Elettronica e Informatica, Facoltà
di Ingegneria, Università degli Studi di Catania, viale A, Doria 6, 95125
Catania, Italy

lfortuna @diees.unict.it

Abstract:

In this paper, we present a new chaotic circuit based on the dynamical system introduced by E. Lorenz in the Journal of the atmospheric sciences, Vol. 20, 130-141(1963). The design and implementation of the circuit are accomplished by using a field programmable analog array (FPAA). The implementation of chaotic Lorenz attractor discussed in this paper is not present in the literature. The experimental results shown in this paper demonstrate that the circuit exhibits pre-chaotic transient and chaotic Lorenz attractor.

Keyword: Field programmable analog array, chaos, chaotic circuits, Lorenz system.

الدوائر الالكترونية المبرمجة تناظرياً- كأساس لنموذج لورنز الفوضوي

¹فاضل رحمة طاهر، ²رمزي سالم علي، ³لويجي فورتونا
^{1,2}جامعة البصرة، كلية الهندسة، قسم الهندسة الكهربائية،
البصرة - العراق

³جامعة كاتانيا، كلية الهندسة، قسم الهندسة الكهربائية و الالكترونية،
كاتانيا-ايطاليا

الخلاصة:

في هذا البحث قُدمت دائرة فوضوية (chaotic circuit) اعتماداً على نموذج ديناميكي مقترح من قبل لورنز في (Journal of the atmospheric sciences, Vol. 20, 130-141(1963)). التصميم والتنفيذ للدائرة أنجز بواسطة المصفوفات التناظرية المبرمجة حقلياً (Field programmable analog arrays-FPAA). جاذب لورنز الفوضوي (chaotic Lorenz attractor) المنفذ في هذا البحث، لم يتم التطرق له سابقاً في الأدبيات ذات العلاقة بالموضوع. النتائج العملية المعروضة وضحت أن الدائرة تُظهر سلوكيات مختلفة مثل transient chaos، chaotic Lorenz attractor.

I- Introduction:

Recently, there has been increasing interest in the designing and implementation of chaotic systems. It has been found that

chaos is useful in many application fields such as engineering, medicine, secure communications, and so on [1-3]. Many chaotic systems have been discovered. The

most famous system is the Lorenz. In 1999, Chen and Ueta developed a new chaotic attractor called Chen attractor by introducing state feedback on the second equation of the Lorenz system[4]. In 2002, Lü and Chen discovered a new chaotic system named Lü system[5]. In 2002, Lü et al. unified the above three chaotic systems into a new chaotic system-unified chaotic system[6].

Theoretical design and circuit implementation of various complex chaotic systems has been a central subject of the real-world applications of various chaos-based technologies and information systems [7,8]. In the last decade a new type of device has emerged, called FPAA (filed programmable analog array), providing analog computation based on the switched capacitor technology. FPAA can be dynamically reconfigured because of this device has a feature that can be used to programmatically change component values and interconnections. Thus, a design modification and new design can be achieved without reset the device. For this reason, FPAA provides useful tool for fast prototyping of analog circuits. Using FPAA, various dynamical systems including different chaotic systems can be implemented at low cost, in a much smaller size and with increased reliability and component stability [9-14].

This paper, introduces an electronic implementation of the dynamical Lorenz system. The electronic implementation is based on a programmable hardware which allows the experimental characterization of the system dynamics with low cost, reconfigurable and rapid experimental setup. AN221E04 FPAA device from Anadigm Inc., is one of the latest production of Anadigm's dynamical programmable Analog Signal Processor (dpASP) series.

The reset of paper is organized as follows: Section II introduces AN221E04 FPAA device. Section III describes the Lorenz model. Circuit implementation will introduced in Section IV. A brief conclusion is given in Section V.

II- Filed Programmable Analog Array(FPAA)

A. FPAA architecture

FPAA's are defined as integrated circuits that can be programmed by the user to implement various analog circuits using circuit building-blocks called configurable analog blocks (CABs) and programmable interconnection network. The CAB consists of primitive analog components whose values and connections can be programmed to implement simple analog functions. Programmable interconnection network routes signals around CABs, and to and from I/O blocks. I/O blocks provide interface between FPAA internal circuits and outside systems [15-17]. Figure 1 shows typical diagram of the FPAA.

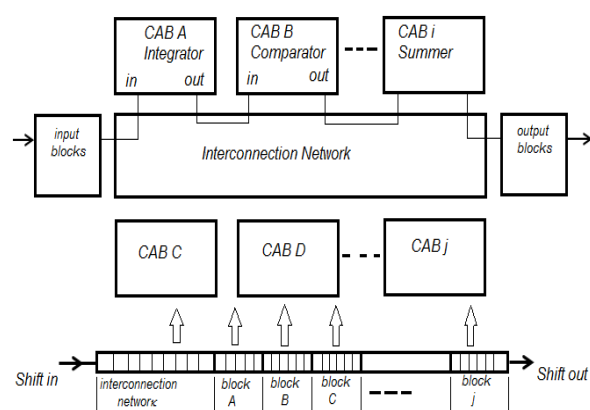


Fig.1:A concept of Field Programmable Analog Array (FPAA).

B. AN221E04 FPAA

The AN221E04 FPAA device is one of latest productions of Anadigm's dynamically programmable analog signal

processor (dpASP) which is integrated on the AN221K04 development board [18-23]. The architecture of AN221E04 is shown in Fig.2. AN221E04 contains a 2×2 matrix of configurable analog blocks (CABs). Configuration data is stored in an on-chip SRAM configuration memory. The four CABs have access to single Look Up Table (LUT), with the capacity 256 Byte, which offers new method of adjusting any programmable element within the device in response to a signal or time base. It can be used to implement arbitrary input-output transfer functions. Analog input signals can be connected from outside world via the four Input/Output cells. Output signals can be routed from within the array out through Input/Output Cells directly. The device can be accept either an external clock or generate its own clock using an on-chip oscillator and an external crystal. The resulting internal clock frequency can be divided down into four synchronized internal switched capacitor clocks. The behavior of the CABs, clocks, signal routing, Input/Output cells, is controlled by the contents of Configuration SRAM. Behind every configuration SRAM bit is a shadow SRAM bit. The shadow SRAM of the AN221E04 devices may be updated without disturbing the currently active analog processing. This allows for on the-fly (dynamic reconfiguration) modification of one or more analog functions.

AN221E04 device allows reconfiguration. While AN221E04 device is operating, the shadow SRAM can be reloaded with values that will sometime later be used to update configuration SRAM. Then, the FPAA can be reprogrammed on-the-fly instantaneously and without interrupting the signal path.

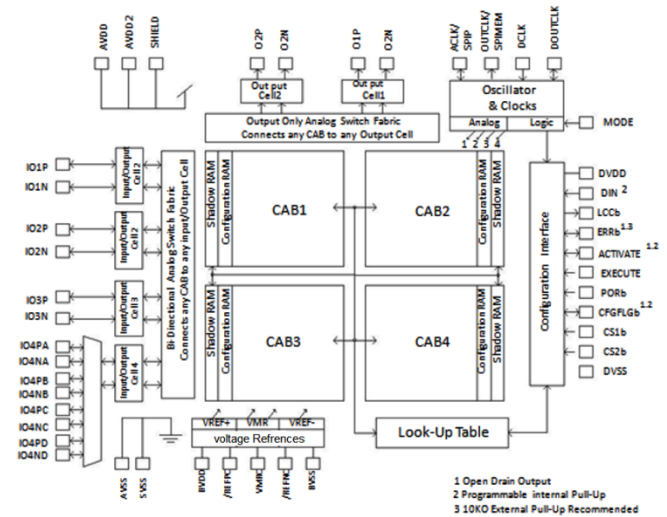


Fig.2:The architecture of AN221E04 FPAA dpASP chip.

This device is based on switched capacitor technology. Switched capacitor circuits are realized with the use of some basic building blocks, such as op-amps, capacitors, switches and non-overlapping clocks. The operation of these circuits is based on the principle of the resistor equivalence of a switched capacitor [24]. This principle is illustrated in Fig.3, where $\Phi 1$ and $\Phi 2$ are the non-overlapping clocks.

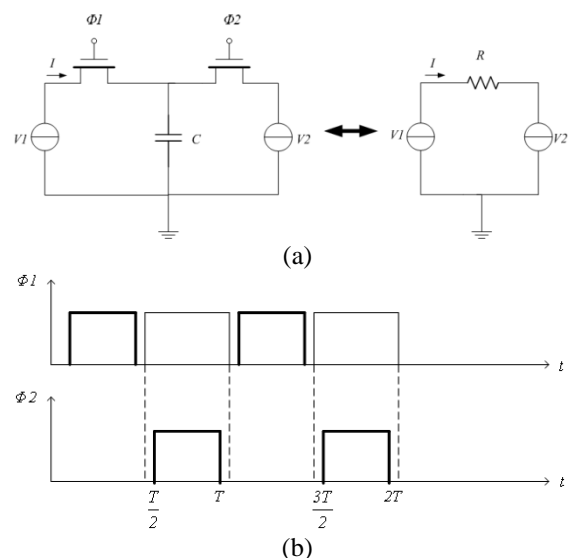


Fig.3: Switched capacitor to resistor equivalence principle.

In Fig. 3(a), if the switches are operate on a two phase clock, a charge will flow through the capacitor C is given by

$$\Delta Q = Q2 - Q1 = C(V2 - V1) \quad \dots(1)$$

So, the current flowing from input to output is

$$I = \frac{\Delta Q}{T} = \frac{(V2-V1)}{T/C} \quad \dots(2)$$

where T is the clock period as shown in Fig. 3(b). From (2), the equivalent current through resistor R , shown in Fig.3(b), flowing from input to output is the same as for I .

III. Lorenz Model

Over fifty years later (1963) Edward Lorenz created the following system [25]:

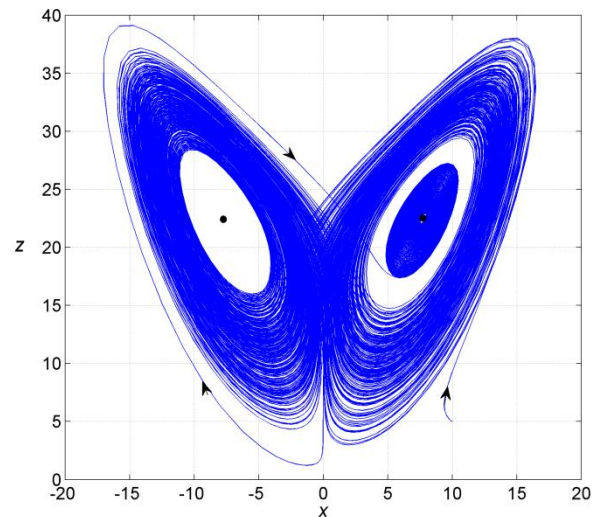
$$\begin{aligned} \dot{x} &= \sigma(y - x) \\ \dot{y} &= rx - y - xz \\ \dot{z} &= xy - bz \end{aligned} \quad \dots(3)$$

Where $(x, y, z) \in \mathbb{R}^3$ and $\sigma > 0, r > 0, b > 0$. This system is called Lorenz system. In fact, this system describes hydrodynamics flow. The variables x, y, z are referring to intensity of convection motion, temperature difference between ascending and descending currents, and distortion of vertical temperature profile, respectively. Follow Lorenz, the fixed parameter values are chosen $\sigma = 10$ and $b = \frac{8}{3}$. To obtain the chaotic behavior from Lorenz system r should be satisfy $r > r_H \approx 24.74$. Where r_H is bifurcation value, and at $r = r_H$ the Hopf bifurcations occur.

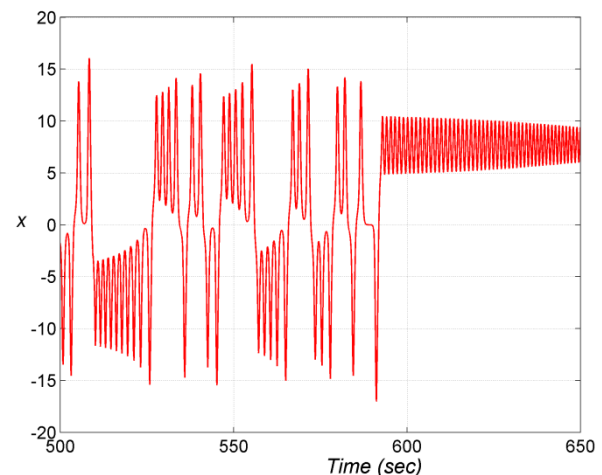
As shown in the simulation trend of Fig.4, the system dynamics exhibit pre-chaotic transient or simply chaos transient for r in the range $r < 24.74$. The trajectory behaves like a chaotic trajectory and then suddenly leaves the chaotic region and for long transient time it finally approaches one of asymptotically stable equilibrium points.

The blue spot shows the trajectory spiraling down to equilibrium point. The time series $x(t)$ shows the same result; initially irregular solution ultimately damps down to equilibrium point as shown in Fig.4(b).

When r passes through r_H , the all equilibrium points become unstable. The solution of Lorenz system (3) appear neither periodic solutions nor convergent



(a)



(b)

Fig.4: Simulation results referring to transient chaos of Lorenz system with parameters $\sigma = 10, b = \frac{8}{3}$, and $r = 23.5$. (a) Phase portrait ($x(t)$ versus $z(t)$). (b) Time series of $x(t)$.

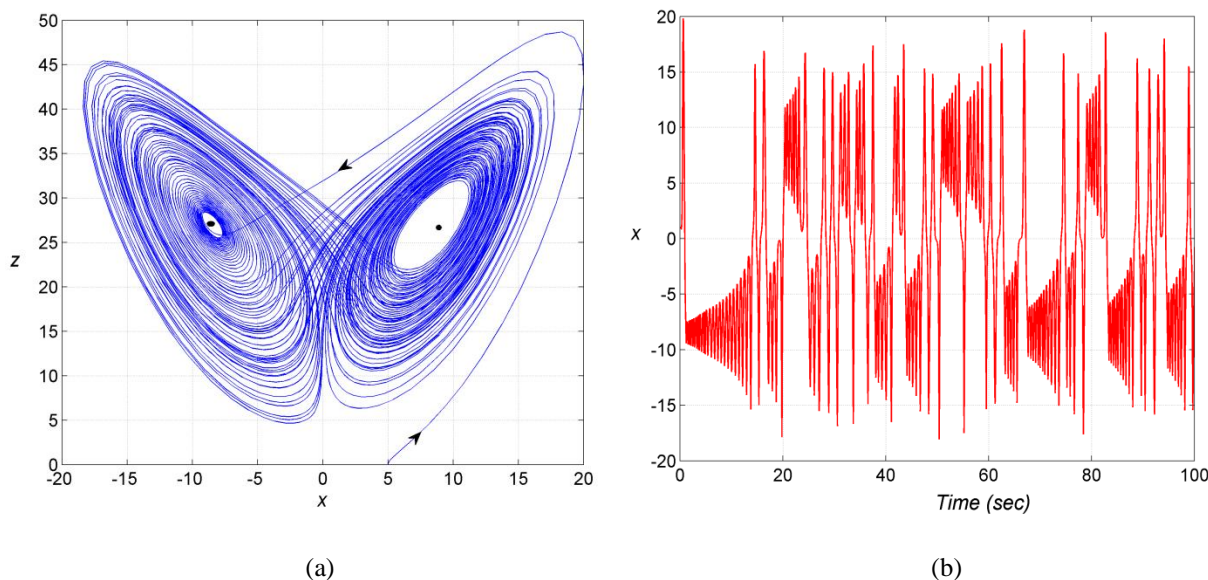


Fig. 5: Simulation results referring to chaotic Lorenz attractor with parameters $\sigma = 10$, $b = \frac{8}{3}$, and $r = 23.5$. (a) Phase portrait ($x(t)$ versus $z(t)$). (b) Time series of $x(t)$.

to equilibrium points. Figure 5 reveals that the solutions of Lorenz system with initial conditions $(x_o, y_o, z_o) = (1, 0, 0)$, close to saddle equilibrium point at the origin when $r = 28$. After an initial transient, the solution settle into an irregular oscillation that never repeats exactly. The motion is aperiodic as shown in Fig.5(b). The trajectory in phase space appears as a butterfly pattern, when $x(t)$ is plotted against $z(t)$ as shown in Fig.5(a). This pattern is repeated endlessly with trajectories being confined to an attractor, so called Lorenz attractor.

IV. FPAAs Implementation of Lorenz Model

The FPAAs used, AN221E04, is included in the development board AN221K04-v4 from Anadigm [23]. The software development tool, AnadigmDesigner2 software, allows the FPAAs to be connected with I/O ports and the desired circuit to be designed using predefined blocks called configurable analog modules

(CAMs), each of which can be used to implement a range of analog functions. CAMs are essentially small building blocks of a circuit such as gains, filters, summations, multiplier, dividers, comparators, oscillators, peak detectors, and rectifiers. A complex circuit can be implemented in a chip simply by selecting, configuring, placing and wiring CAMs [26].

The dynamic range that allowed by FPAAs device is ± 2 , a rescaling process may be required according to results. After modeling system implementation in FPAAs software tool, this model is downloaded to FPAAs chip on the development board via serial interface.

Since the dynamic range of the Lorenz system variables (peak-to-peak oscillations equal to 40 units) is larger than that allowed by FPAAs device, a rescaling was necessary to fit the dynamic range of FPAAs internal voltages. The following rescaling factors are chosen:

$$X = \frac{x}{k_x}, Y = \frac{y}{k_y}, Z = \frac{z}{k_z}, \quad \dots(4)$$

with $k_x = 5, k_y = 10, k_z = 40$. By applying this rescaling the Lorenz system (3) becomes:

$$\begin{aligned} \dot{X} &= \sigma \left(\frac{k_y}{k_x} Y - X \right) \\ \dot{Y} &= r \frac{k_x}{k_y} X - Y - \frac{k_x k_z}{k_y} XZ \\ \dot{Z} &= \frac{k_x k_y}{k_z} XY - bZ. \end{aligned} \quad \dots(5)$$

All circuit parameters are programmable and their values cannot be accurately fixed. They are implemented with some parameter tolerances. Moreover, introducing further parameters in system (5) was necessary in order to overcome the inaccuracies due to the programmable device, so the model implemented on the analog device can be written as follows:

$$\begin{aligned} \dot{X} &= \alpha \left(\frac{k_y}{k_x} Y - X \right) \\ \dot{Y} &= G_1 r \frac{k_x}{k_y} X - G_2 Y - G_3 \frac{k_x k_z}{k_y} XZ \\ \dot{Z} &= G_4 \frac{k_x k_y}{k_z} XY - G_5 bZ. \end{aligned} \quad \dots(6)$$

The parameters $G_i (i = 1, 2, \dots, 5)$ are experimentally tuned to compensate these inaccuracies. The circuit schematic is shown in Fig.6. The integration constant for integrators is $0.0025 \text{ } 1/\mu\text{s}$. The sum block SD1 sums the two terms need for X variables. The blocks SD2 and SD3 are summing the terms needed for Y and Z , respectively. The two nonlinear terms are implemented by the two multiplier blocks M1 and M2. A reconstruction filter was selected at the OutputCell2 to allow the removal of higher frequency components that are introduced by sampling behavior of the SC CAMs. The filter corner frequency (76-470 kHz) should be set based on the signal frequency and sample clock rate. This filter parameter should be far enough

above the highest frequency component of the signal so that the signal

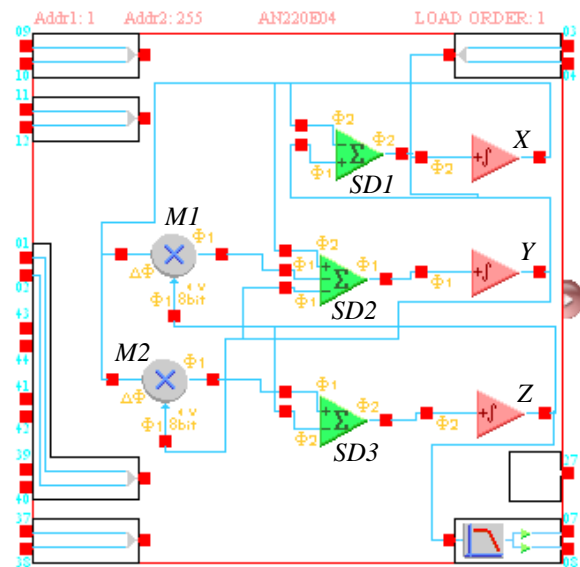


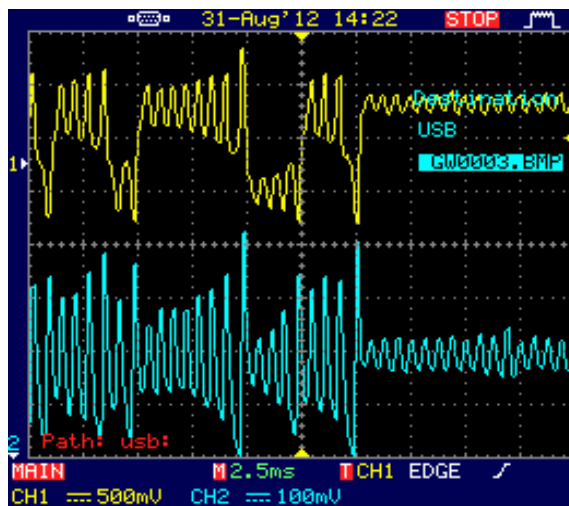
Fig.6: AnadigmDesigner2 screenshot of FPA implementation circuit scheme of Lorenz system.

is not attenuated and far enough below the sampling frequency set by the sampling clocks so that the higher frequency components are attenuated.

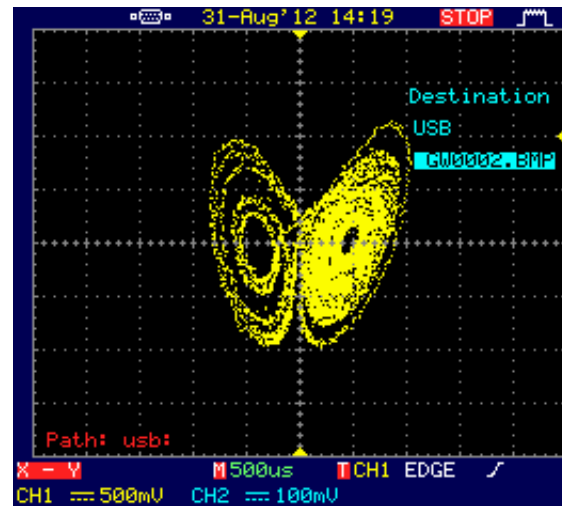
Several experiments have been performed to investigate the behavior of the circuit. The behavior of the circuit has been investigated for different parameter values. The experimental observations confirm the numerical simulation results of the pre-transient chaos followed by chaotic Lorenz attractor. Figures 7 and 8 refer to two different sets of parameters leading to chaotic behavior. The time waveforms of the signal X and Z are shown in Figs. 7(a) and 8(a), while the phase portrait of X versus Z is shown in Figs.7(b) and 8(b). The behavior show an initially aperiodic oscillations ultimately damp down to a stable equilibrium point. The phase portrait of X versus Z shows the same result; at first the oscillation seems to be tracing out a strange attractor and the trajectory behaves like chaotic oscillation, but eventually it leaves the chaotic region and stays on the right and spirals down for long transient

time around the stable equilibrium point.

The yellow spot shows

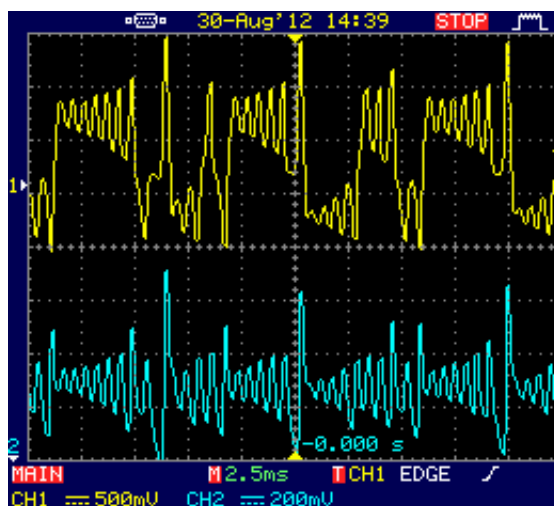


(a)

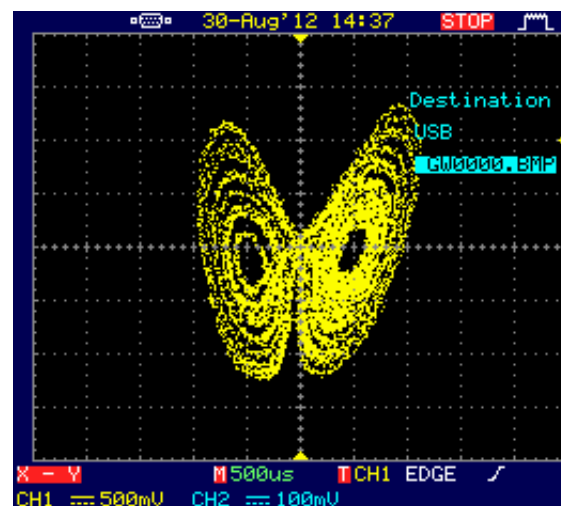


(b)

Fig.7: Experimental observations referring to the transient chaos of the circuit with parameters $G_1 = 0.367$, $G_2 = 0.2$, $G_3 = 1.1$, $G_4 = 1.6$, and $G_5 = 1.2$. (a) Time domain waveforms ($X(t)$ is channel 1, $Z(t)$ is channel 2). (b) Phase portrait ($X(t)$ versus $Z(t)$).



(a)



(b)

Fig.8: Experimental observations referring to the chaotic behavior of the circuit with parameters $G_1 = 0.421$, $G_2 = 0.2$, $G_3 = 1.23$, $G_4 = 1.6$, and $G_5 = 1.2$. (a) Time domain waveforms ($X(t)$ is channel 1, $Z(t)$ is channel 2). (b) Chaotic Lorenz butterfly attractor ($X(t)$ versus $Z(t)$).

the oscillation spirals down to equilibrium point as shown in Fig.8(b).

Increasing G_1 to approximately 5 leads to Hopf bifurcation and all the equilibrium points become unstable. The oscillation appears neither periodic nor convergent to equilibrium points. The chaotic oscillation appears as a butterfly pattern. The occurrence of chaotic behavior from

Lorenz system can be clearly seen from Fig.8. Both phase portrait and time domain waveforms indicate the chaotic oscillation. This is confirmed by Fast Fourier Transform (FFT) from the scope, the wideband nature of the spectra in Fig. 9 shows the chaos. A chaotic waveform has a broadband power spectrum.

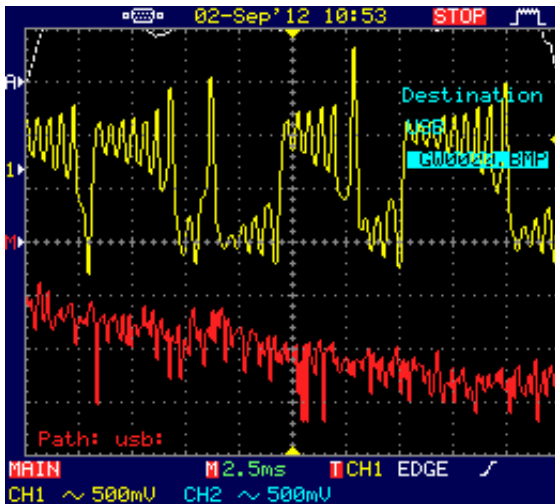


Fig.9: Time domain waveforms $X(t)$ and corresponding Fast Fourier Transform (FFT), 20 dB/division.

V-Conclusion:

In this paper, a programmable analog circuit implementing Lorenz system dynamics has been introduced. The design and implementation of the circuit are based

on an analog programmable device, the FPAA, which provides the possibility of experimentally exploring the different dynamics of Lorenz system in a rapid and low cost way.

The pre-chaotic transient and chaotic oscillation typically shown by Lorenz system has been experimentally observed. The circuit has been experimentally investigated under different sets of parameters values. The observation results demonstrate very good agreement between numerical simulation and experiment. This is a motive, for future, to extend practical researches and demonstrations over to other types of systems.

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